12 April 2005   **Final Exam**   **Time Allocated: 3 hours**

NAME:_____   STUDENT NUMBER: _____

NOTE: This is a closed book exam. No electronic devices, notes or communication with other students are allowed. This exam is comprised of 9 pages.

PART A: Each of the 18 problems in part A counts 3 marks. For each problem, provide your answer in the space available.

1.   If $Cos^{-1}(\ln(x+1)) = \pi/3$, then $x = $_____

2.   If $y = \int_{4}^{x^2} \dfrac{1 + e^t}{\sqrt{t}}\,dt$, then, at $x = 2$, $\dfrac{dy}{dx} = $ _____

3.   If $y = (\sinh 2x)e^{Sin^{-1}x}$, then $\dfrac{dy}{dx} = $_____

4.   If $y = Tan^{-1}(1 + \cosh x)$, then $\dfrac{dy}{dx} = $_____

5. $\int_{0}^{1}(e^{3x} + \cos(x-1) + x^3)\,dx = $_____

6. $\int \dfrac{Sin^{-1}x}{\sqrt{1 - x^2}}\,dx = $ _____

7. $\int \dfrac{2\sin x}{1 + \cos x}\,dx = $_____

8. $\int \tan^2(3x)\,dx = $_____

9. $\int \dfrac{(1 + \sqrt{x})^{10}}{\sqrt{x}}\,dx = $ _____

NAME:_____ STUDENT NUMBER: _____

10. $\int \dfrac{6}{x^2 - 9}\, dx =$ _____

11. $\int x \sin(4x)\, dx =$ _____

12. A surface of revolution is generated by rotating the arc of the curve $y = \cos x$ from the point $(0,1)$ to the point $(\pi/2, 0)$ about the x-axis. The area of this surface is equal to the integral

_____

Problems 13, 14, and 15 are concerned with the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 1$.

13. The area of the above region is_____

14. The volume of the solid of revolution generated by revolving this region about the x-axis is_____

15. The volume of the solid of revolution generated by revolving this region about the y-axis is_____

16. Does the improper integral $\displaystyle\int_{1}^{\infty} \dfrac{e^{1/x}}{x^2}\, dx$ converge? If it does, state the value to which the integral converges. If it does not, state why._____

17. The polar equation $r = \dfrac{4}{2\cos\theta - \sin\theta}$ may be expressed in rectangular coordinates in the form $y = f(x)$ where $f(x) =$_____

18. If $x = t - t^2$ and $y = t - t^3$, find $\dfrac{dy}{dx}$ at $t = 1$._____

NAME:_____   STUDENT NUMBER: _____

PART B: The marks for problems 19 to 24 are shown in the left-hand margin. Provide your answer in the space available.

[6]   19.   For the region bounded by $y = \dfrac{16}{x^2}$, $y = 1$ and $y = 4$:

(a)   Show that the area of this region is 16 square units.

(b)   Find the centroid of this region.

(c)   Find the volume of the solid of revolution if this region is revolved about the line $y = -1$.

NAME:_____ STUDENT NUMBER: _____

[6]   20.   For the definite integral $\int_0^2 4^{2x^2-3x}dx$:

(a) Use the Trapezoidal Rule with $n = 4$ subintervals to estimate the value of this integral. Express your answer in as an integer or fraction.

(b) Use Simpson's Rule with $n = 4$ subintervals to estimate the value of this integral. Express your answer in as an integer or fraction.

NAME:_____ STUDENT NUMBER: _____

[6]   21.   A tank initially holds 2000 litres of a salt-water solution in which 10 kilograms of salt are dissolved. Brine that contains 0.04 kilograms of salt per litre of water enters the tank at a rate of 25 litre per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate.

(a)   Determine the function of time, $S(t)$, that yields the amount of salt in the tank for $t \geq 0$.

(b)   Given that $\ln 2 \approx 0.7$, determine the time when the amount of salt would be 45 kilograms.

NAME:_____   STUDENT NUMBER: _____

[6]   22. The face of a dam is in the shape of an isosceles trapezoid where the bottom and top of the dam
          are 20 meters and 30 meters respectively, and the vertical height is 10 meters (diagram below).
          If the water level is to a height 4 meters below the top of the dam, determine the total force due
          to water pressure on this face. Express your answer in terms of the water density ρ and the
          gravitational constant g.
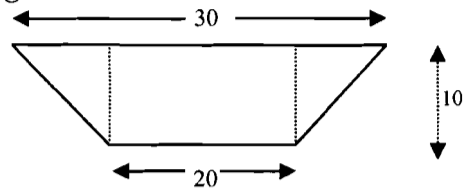
NAME:_____ STUDENT NUMBER: _____

[6] 23. A cable, attached to a pulley on a platform, hangs a vertical distance of 6 meters with an additional 4 meters of this cable tied around a container full of concrete mixture sitting on the ground. The cable has a mass of 0.5 kilograms per meter and the mass of the container and concrete mixture is 98 kilograms. Find the work done in raising the container 5 meters. Express your answer in terms of the gravitational constant g.

NAME:_____   STUDENT NUMBER: _____

[16]   24. Evaluate the following four integrals.

(a) $\int (\tan^3 x)(\sec^3 x)\,dx$

(b) $\int \dfrac{dx}{\sqrt{4x - x^2}}\,dx$

NAME:_____ STUDENT NUMBER: _____

(c) $\int (e^{2x} \sin x) \, dx$

(d) $\int \dfrac{x^2 + 2}{x^3 + 4x^2 + 4x} \, dx$

** The End **